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Research Article

Exact Solutions for a Nonlinear Dynamical System in a New Double-Chain Model of DNA Using the Modified Simple Equation Method

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Abstract: In this article, we apply the modified simple equation method to find the exact solutions with parameters of the general nonlinear dynamical system in a new double-chain model of DNA. When the parameters are assigned special values, the solitary wave solutions are derived from the exact solutions. Comparison between our results and the well-known results is given.

Keywords: Nonlinear dynamics of DNA model; Exact solutions; Modified simple equation method; Double-chain model of DNA.

1. Introduction

The investigation of the traveling wave solutions for nonlinear partial differential equations plays on important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. Because of the increased concentration in the theory of solitary waves, a large variety of analytic and computational methods have been established in the analysis of the nonlinear models (see [1-19]). An attractive nonlinear model for the nonlinear science is the deoxyribonucleic acid (DNA). The dynamics of DNA molecules is one of the most fascinating problems of modern biophysics because it is at the basis of life. The DNA structure has been studied during last decades. The investigation of DNA dynamics has successfully predicted the appearance of important nonlinear structures. It has been shown that the nonlinearity is responsible for forming localized waves. These localized waves are interesting because they have the capability to transport energy without dissipation [20-28]. In Ref. [27,28], it is given that a new double-chain model of DNA consists of two long elastic homogeneous strands which represent two polynucleotide chains of the DNA molecule, connected with each other by an elastic membrane representing the hydrogen bonds between the base pair of the two chains. Under some appropriate approximation, the new double-chain model of DNA can be described by the following two general nonlinear dynamical system:

$$u_{tt} - c_1^2 u_{xx} = \lambda_1 u + \gamma_1 uv + \mu_1 u^3 + \beta_1 uv^2, \tag{1}$$

$$v_{tt} - c_2^2 v_{xx} = \lambda_2 v + \gamma_2 u^2 + \mu_2 u^2 v + \beta_2 v^3 + c_0, \tag{2}$$

where

$$\begin{aligned} &c_1 = \pm\sqrt{\frac{Y}{\rho}}; \quad c_2 = \pm\sqrt{\frac{F}{\rho}}; \quad \lambda_1 = \frac{-2\mu}{\rho\sigma h}(h - l_0); \\ &\lambda_2 = \frac{-2\mu}{\rho\sigma}, \quad \gamma_1 = 2\gamma_2 = \frac{2\sqrt{2}\mu l_0}{\rho\sigma h^2}; \quad \mu_1 = \mu_2 = \frac{-2\mu l_0}{\rho\sigma h^3}; \\ &\beta_1 = \beta_2 = \frac{4\mu l_0}{\rho\sigma h^3}; \quad c_0 = \frac{\sqrt{2}\mu(h - l_0)}{\rho\sigma}, \end{aligned} \tag{3}$$

where ρ, σ, Y and F denote respectively the mass density, the area of transverse cross-section, the Young's modulus and the tension density of each strand; μ is the rigidity of the elastic membrane; h is the distance between the two strands, and l_0 is the height of the membrane in the equilibrium position. In Eqs. (1) and (2), u is the difference of the longitudinal displacements of the bottom and top strands, while v is the difference of the transverse displacements of the bottom and top strands.

The objective of this article is to construct the exact solutions of the dynamical system (1) and (2) using the modified simple equation method. The rest of this article is organized as follows: In Sec.2, we describe the modified simple equation method. In Sec.3, we solve the system (1) and (2) using this method. In Sec.4, some conclusions and discussions are given.

2. Description of the Modified Simple Equation Method

Consider a nonlinear evolution equation in the form:

$$F(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0, \tag{4}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [11-13] :

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \quad \xi = kx + \omega t, \tag{5}$$

where k and ω are constants, to reduce Eq.(4) to the ODE:

$$P(u, u', u'', \ldots) = 0, \tag{6}$$

where P is a polynomial in $u(\xi)$ and its total derivatives with respect to ξ, while

$' = d/d\xi$

Step 2. We assume that Eq.(6) has the formal solution

$$u(\xi) = \sum_{k=0}^{N} a_k \left[\frac{\psi'(\xi)}{\psi(\xi)}\right]^k \tag{7}$$

where $a_k (k = 0,1,..., N)$ are constants to be determined later, such that $a_N \neq 0$. The function $\psi(\xi)$ is an unknown function to be determined later such that $\psi'(\xi) \neq 0$.

Step 3: We determine the positive integer N in Eq. (7) by balancing the highest order derivatives and the nonlinear terms in Eq. (6).

Step 4: We substitute (7) into (6), we calculate all the necessary derivatives $u', u'', u''', ...$ of the unknown function $u(\xi)$ and then we account the function $\psi(\xi)$. As a result of this substitution, we obtain a polynomial of $\psi^{-j}, (j = 0,1,...)$. In this polynomial, we gather all the terms of the same power of $\psi^{-j}, (j = 0,1,...)$, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_i and $\psi(\xi)$. Consequently, we can obtain the exact solutions of Eq.(4) .

3. Applications

In this section, we apply the modified simple equation method to find the exact solutions of the dynamical system (1) and (2). To this end, we first introduce the transformation

$$v = au + b, \tag{8}$$

where a and b are constants, to reduce Eqs. (1) and (2) to the following system of equations:

$$u_{tt} - c_1^2 u_{xx} = u^3(\mu_1 + \beta_1 a^2) + u^2(2\beta_1 ab + a\gamma_1) + u(\lambda_1 + b\gamma_1 + \beta_1 b^2), \tag{9}$$

and

$$u_{tt} - c_2^2 u_{xx} = u^3(\mu_2 + \beta_2 a^2) + u^2\left(\frac{\gamma_2}{a} + \frac{\mu_2 b}{a} + 3\beta_2 ab\right) + u(\lambda_2 + 3\beta_2 b^2) + \frac{\lambda_2 b}{a} + \frac{\beta_2 b^3}{a} + \frac{c_0}{a}. \tag{10}$$

Comparing Eqs.(9) and (10) and using (3) we deduce that $b = \frac{h}{\sqrt{2}}$ and $F = Y$. Now Eqs. (9) and (10) can be written as

$$u_{tt} - c_1^2 u_{xx} - Au^3 - Bu^2 - Cu = 0 \tag{11}$$

where

$$A = \left(\frac{-2\alpha}{h^3} + \frac{4a^2\alpha}{h^3}\right); \quad B = \frac{6\sqrt{2}a\alpha}{h^2}, \quad C = \left(\frac{-2\alpha}{l_0} + \frac{6\alpha}{h}\right), \quad \alpha = \frac{\mu l_0}{\rho\sigma}, \quad c_1^2 = \frac{Y}{\rho}. \tag{12}$$

The wave transformation (5) of Sec.2, reduces Eq.(11) to the following ODE:

$$(\omega^2 - k^2c_1^2)u'' - Au^3 - Bu^2 - Cu = 0, \tag{13}$$

where $\omega^2 - k^2c_1^2 \neq 0$. Balancing u'' with u^3 yields $N = 1$. Consequently, we have the formal solution:

$$u(\xi) = a_0 + a_1\left[\frac{\psi'(\xi)}{\psi(\xi)}\right], \tag{14}$$

where a_0 and a_1 are constants to be determined, such that $a_1 \neq 0$. The unknown function $\psi(\xi)$ is also to be determined such that $\psi'(\xi) \neq 0$. From (14), it is easy to see that

$$u'(\xi) = a_1\left[\frac{\psi''(\xi)}{\psi(\xi)} - \frac{\psi'^2(\xi)}{\psi^2(\xi)}\right], \tag{15}$$

$$u''(\xi) = a_1\left[\frac{\psi'''(\xi)}{\psi(\xi)} - 3\frac{\psi'(\xi)\psi''(\xi)}{\psi^2(\xi)} + 2\frac{\psi'^3(\xi)}{\psi^3(\xi)}\right]. \tag{16}$$

Substituting Eqs.(14),(16) into Eq.(13) and equating all the coefficients of $\psi^0, \psi^{-1}, \psi^{-2}, \psi^{-3}$ to be zero, we obtain

$$a_0\left[Aa_0^2 + Ba_0 + C\right] = 0, \tag{17}$$

$$\left(\omega^2 - k^2c_1^2\right)\psi''' - \left(3a_0^2A + 2a_0B + C\right)\psi' = 0, \tag{18}$$

$$3\left(\omega^2 - k^2c_1^2\right)\psi'' + (3a_0A + B)a_1\psi' = 0, \tag{19}$$

$$2\left(\omega^2 - k^2c_1^2\right) - Aa_1^2 = 0. \tag{20}$$

On solving Eqs. (17)and(20) we have the results:

$$a_0 = 0, \quad Aa_0^2 + Ba_0 + C = 0, \quad a_1 = \pm\sqrt{\frac{2}{A}\left(\omega^2 - k^2c_1^2\right)}.$$

Let us now discuss the following cases:

Case 1. If $a_0 = 0$, then Eqs. (18) and (19) reduce to

$$\left(\omega^2 - k^2c_1^2\right)\psi''' - C\psi' = 0, \tag{21}$$

$$3\left(\omega^2 - k^2c_1^2\right)\psi'' + Ba_1\psi' = 0. \tag{22}$$

From (21) and (22) we have

$$\psi'''/\psi'' = \frac{-3C}{Ba_1}. \tag{23}$$

Consequently, we deduce that

$$\psi' = \frac{-3Aa_1\alpha_1}{2B}\exp\left[\frac{-3C}{Ba_1}\xi\right], \tag{24}$$

$$\psi = \alpha_2 + \frac{Aa_1^2\alpha_1}{2C}\exp\left[\frac{-3C}{Ba_1}\xi\right], \tag{25}$$

where α_1 and α_2 are arbitrary constants of integration. Now, the exact solution of Eq.(11)) in this case has the form:

$$u(\xi) = \frac{-3C}{B}\left\{\frac{\exp\left[\frac{-3C}{Ba_1}(\xi+\xi_0)\right]}{\alpha_2+\exp\left[\frac{-3C}{Ba_1}(\xi+\xi_0)\right]}\right\}, \tag{26}$$

where, $\exp(-\frac{3C}{Ba_1}\xi_0) = \alpha_1\left(\omega^2 - k^2c_1^2\right)$ and ξ_0 is constant. If we set $\alpha_2 = \pm 1$ in (26) then we have respectively the following solitary wave solutions:

$$u_1 = \frac{-3C}{2B}\left\{1-\tanh\left[\frac{3C}{2Ba_1}(\xi+\xi_0)\right]\right\}, \tag{27}$$

$$u_2 = \frac{-3C}{2B}\left\{1-\coth\left[\frac{3C}{2Ba_1}(\xi+\xi_0)\right]\right\}. \tag{28}$$

Case 2. If $a_0 \neq 0$, then we deduce from Eqs.(18) and (19) that

$$\psi'''/\psi'' = \frac{-3a_0(B+2a_0A)}{a_1(B+3a_0A)}, \tag{29}$$

where, $a_0 = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$. Consequently, we conclude that

$$\psi' = \frac{-3\beta_1\left(\omega^2-k^2c_1^2\right)}{a_1(B+3a_0A)}\exp\left[\frac{-3a_0(B+2a_0A)}{a_1(B+3a_0A)}\xi\right], \tag{30}$$

$$\psi = \beta_2 + \frac{\beta_1\left(\omega^2-k^2c_1^2\right)}{a_0(B+2a_0A)}\exp\left[\frac{-3a_0(B+2a_0A)}{a_1(B+3a_0A)}\xi\right], \tag{31}$$

where β_1 and β_2 are arbitrary constants of integration. Now, the exact solution of Eq.(11) in this case has the form:

$$u(\xi) = a_0 - \frac{3a_0(B+2a_0A)}{(B+3a_0A)}\left\{\frac{\exp\left[\frac{-3a_0(B+2a_0A)}{a_1(B+3a_0A)}(\xi+\xi_0)\right]}{\beta_2+\exp\left[\frac{-3a_0(B+2a_0A)}{a_1(B+3a_0A)}(\xi+\xi_0)\right]}\right\}, \quad (32)$$

where, $\exp\left[\frac{-3a_0(B+2a_0A)}{a_1(B+3a_0A)}\xi_0\right] = \frac{\beta_1\left(\omega^2-k^2c_1^2\right)}{a_0(B+2a_0A)}$ and ξ_0 is a constant. If we set

$\beta_2 = \pm 1$ in (32) then we have respectively the following solitary wave solutions:

$$u_1 = a_0 - \frac{3a_0(B+2a_0A)}{2(B+3a_0A)}\left\{1-\tanh\left[\frac{3a_0(B+2a_0A)}{2a_1(B+3a_0A)}(\xi+\xi_0)\right]\right\}, \quad (33)$$

$$u_2 = a_0 - \frac{3a_0(B+2a_0A)}{2(B+3a_0A)}\left\{1-\coth\left[\frac{3a_0(B+2a_0A)}{2a_1(B+3a_0A)}(\xi+\xi_0)\right]\right\}. \quad (34)$$

4. Some Conclusions and Discussions

The modified simple equation method presented in this article has been employed to find the exact solution (26) and the solitary wave solutions (27) and (28) in the case $a_0 = 0$ as well as the exact solution (32) and the solitary wave solutions (33) and (34) in the case $a_0 \neq 0$ for the nonlinear dynamics equation(11). Consequently, we can find the solution v by the aid of Eq.(8). Now, the solution of the system (1) and (2) has been found. Alka et al [28] have investigated the model (11) using the elliptic equation method and found the following wave solutions:

$$u(\xi) = \frac{-\sqrt{2}ah}{(2a^2-1)}\left[1\pm\tanh\left(\sqrt{\frac{2a^2\mu l_0}{\rho\sigma h(2a^2-1)}}\xi\right)\right], \quad (35)$$

where the plus or minus sign stands for anti-kinks or kinks respectively, while ξ is given by

$$\xi = (x-\sqrt{2}c_1t)/c_1, \quad a^2 > \frac{1}{2}. \quad (36)$$

On comparing our results (27) with the well-known result (35) we deduce after some reductions that they are equivalent. On the other hand, in this article we have obtained further results (26), (28), (32), (33) and (34) which look new and they are not reported elsewhere. Finally, all solutions obtained in this article have been checked with the Maple by putting them back into the original equation (11).

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