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Research Note

On the Pandey-Sharma Spacetime

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Abstract: We exhibit that the Pandey-Sharma's spacetime is a counterexample for a conjecture of Boyer-Plebañski and for a theorem of Sen.

Keywords: Boyer-Plebañski's conjecture; Sen's theorem; 4-space of class one; Pandey-Sharma spacetime.

Pandey and Sharma, 1981 considered the spacetime with spherical symmetry (Modak, 1984):

$$ds^2 = dr^2 + r^2(d\theta^2 + sin^2\theta\, d\varphi^2) - \left(1 + B(t)\, r^2\right)^2 dt^2, \qquad (1)$$

for a conformally flat perfect fluid distribution with zero density, where $B(t) \neq 0$ is an arbitrary function. The corresponding non-null components of Riemann and Ricci tensors, and the scalar curvature are given by [$(x^k) = (r, \theta, \varphi, t)$]:

$$R_{44} = -3R_{1414} = -6B(1 + Br^2), \qquad R_{33} = sin^2\theta\, R_{22} = r^2 sin^2\theta\, R_{11}, \qquad (2)$$

$$R = 6R_{11} = \frac{12\, B}{1 + Br^2}, \qquad R_{3434} = sin^2\theta\, R_{2424} = r^2 sin^2\theta\, R_{1414},$$

with the Weyl tensor equals to zero.

On the other hand, we have the Boyer and Plebañski, 1993 conjecture:

"If R_4 is conformally flat with spherical symmetry, then it has class one", (3)

therefore (1) should be of class one, however, Pandey and Sharma, 1981 proved that this is false, that is, (1) does not accepts local and isometric embedding into E_5. In other words, (1) is a counterexample for the Boyer-Plebañski's conjecture; besides, this spacetime also shows the insufficiency of the conditions of Karmarkar, 1948 because it verifies such conditions but has not class one.

The theorem of Sen, 1966 is given by:

"If R_4 is conformally flat and its curvature tensor has the structure:

$$R_{ijkm} = E(R_{ik}R_{jm} - R_{im}R_{jk}) + F(g_{ik}g_{jm} - g_{im}g_{jk}), \qquad (4)$$

with $E \neq 0$ and F scalars, then R_4 has class one".

The quantities (2) permit to show that the Riemann tensor verifies the expression (4) for $E = -\frac{3}{R}$ and $F = \frac{R}{12}$, however, (1) is not of class one and thus it is a counterexample for the Sen's theorem.

The perfect fluid for (1) has zero density, but it is interesting to note that Gupta and Pandey, 1976 constructed spherical symmetric conformally flat metrics of class one for perfect fluid distribution with density ≠ 0.

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