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Research Article

Super Vertex Mean Labeling of Cyclic Snakes

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Abstract:A Super Vertex Mean labeling f of a (p, q) - graph $G(V, E)$ is defined as an injection from E to the set $\{1, 2, 3, \cdots, p+q\}$ that induces for each vertex v the label defined by the rule $f^v(v) = \text{Round}\left(\frac{\sum_{e\in E_v} f(e)}{d(v)}\right)$, where E_v denotes the set of edges in G that are incident at the vertex v, such that the set of all edge labels and the induced vertex labels is $\{1, 2, 3, \cdots, p+q\}$. All the cycles, C_n, $n \geq 3$ and $n \neq 4$ are Super Vertex Mean graphs. Our attempt in this paper is to show that all the cyclic snakes are also Super Vertex Mean graphs.

Keywords: Super Vertex Mean label; Cyclic Snakes; Types of cyclic snakes.

1 Introduction

A graph is an ordered pair $(V(G), E(G))$, consisting of a finite non empty set $V(G)$ of objects called points or vertices and a set $E(G)$ of 2-element subsets of $V(G)$, known as edges. The sets $V(G)$ and $E(G)$ are the vertex set and edge set respectively. The cardinality of $V(G)$is the order of the graph G and is often denoted by $|V(G)|= p$, and that of $E(G)$ is the size of G and is denoted by $|E(G)| = q$. A graph of order p and size q is often called a (p, q) - graph.

A labeling of a graph G is an assignment of labels either to the vertices or edges. If the domain is the set of vertices, then the labeling is known as vertex labeling. A vertex labeling of a graph G is an assignment f of labels to the vertices of G that induces a label for each edge uv depending on the vertex labels. Otherwise it is edge labeling. An edge labeling of a graph G is assignment f of labels to the edges of G that induces a label for each vertex v depending on the edge labels. There are varieties of vertex as well as edge labeling that are already in the literature.

Graceful labeling was first introduced by Rosa in 1967 [12], though he called this labeling as β- valuation. Several years later, Golomb [3] studied the same and named it graceful labeling. Later Acharya introduced Super graceful labeling and Singh and Devraj brought in the concept of Triangular graceful labeling. Acharya and Germina [1] further introduced an edge analogue of graceful labeling and named it as vertex graceful numbering.

Harmonious labeling is one of the fundamental labeling introduced by Graham and Solane [4] in 1980 in connection with their study on error correction code. Mean labeling was introduced by Somasundaram and Ponraj [14]. Let $G = (V, E)$ be a simple graph with p vertices and q edges. A mean labeling f is an injection from V to the set $\{0, 1, 2, \cdots, q\}$ that induces for each edge uv the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ such that the set of edge labels is $\{1, 2, \cdots, q\}$. A graph that accepts a mean labeling is known as mean graph.

A super mean labeling f is an injection from V to the set $\{1, 2, \cdots, p + q\}$ that induces for each edge uv the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ such that the set of all vertex labels and the induced edge labels is $\{1, 2, \cdots, p + q\}$. This concept was introduced by D.Ramya et al.[10]. Some results on mean labeling and super mean labeling are given in [5], [6], [9], [10], [11], [13], [14] and [15].

Lourdusamy and Seenivasan [5] introduced vertex mean labeling as an edge analogue of mean labeling as follows: A vertex mean labeling of a (p, q) - graph $G(V, E)$ is defined as an injection $f : E \longrightarrow \{0, 1, \cdots, q*\}$, $q* = \max(p, q)$ such that the injection $f : V \to N$ defined by the rule $f^v(v) = \text{Round}\left(\frac{\sum_{e \in E_v} f(e)}{d(v)}\right)$ satisfies the property that $f^v(V) = \{f^v(u) : u \in V\} = \{1, 2, ..., p\}$, where E_v denotes the set of edges in G that are incident at v and N denotes the set of all natural numbers. A graph that has a vertex mean labeling is called a vertex mean graph or $V-$ mean graph.

Continuing on the same line and inspired by the above mentioned concepts, Lourdusamy et al. [7] brought in a new extension of mean labeling, called Super vertex mean labeling of graphs.

2 Super Vertex Mean Labeling

Definition 2.1 *A Super Vertex Mean labeling f of a (p, q) - graph $G(V, E)$ is defined as an injection from E to the set $\{1, 2, 3, \cdots, p + q\}$ that induces for each vertex v the label defined by the rule $f^v(v) = Round\left(\frac{\sum_{e \in E_v} f(e)}{d(v)}\right)$, where E_v denotes the set of edges in G that are incident at the vertex v, such that the set of all edge labels and the induced vertex labels is $\{1, 2, 3, \cdots, p + q\}$.*

A graph that accepts super vertex mean labeling is called a Super Vertex Mean (hereafter, SVM) graph.

The following results have already been proved in [7] and [8]. We use them for our further study on the Super Vertex Mean behaviour of cyclic snakes. Before entering into the results, we define the term Cyclic Snakes.

Definition 2.2 *A kC_n – snake has been defined as a connected graph in which all the blocks are isomorphic to the cycle C_n and the block-cut point graph is a path P, where P is the path of minimum length that contains all the cut vertices of a kC_n –snake. Barrientos [13] has proved that any kC_n – snake is represented by a string $s_1, s_2, s_3, \cdots, s_{k-2}$ of integers of length $k-2$, where the i^{th} integer, s_i on the string is the distance between i^{th} and $i+1^{th}$ cut vertices along the path, P, from one extreme and is taken from $S_n = \{1, 2, 3, \cdots, \lfloor \frac{n}{2} \rfloor\}$.*

Remark 2.3 *The strings obtained for both the extremes are assumed to be the same. In this paper we consider only those Cyclic snakes with $s_i = 1$, for $1 \leq i \leq k-2$.*

3 Known Results

3.1 Result 1: All the Cycles Except C_4 are SVM Graphs [8]

Case 1: $n \equiv 1(mod\ 2)$.

Let C_n be an odd cycle with n vertices. Let $\{e_1, e_2, \cdots, e_n\}$ be the edge set and $\{v_1, v_2, \cdots, v_n\}$ be the vertex set of C_n such that $e_i = v_i v_{i+1}, 1 \leq i \leq n-1$ and $e_n = v_n v_1$.

Let $n = 2r+1$. The edges of C_n are labeled as follows:

$$f(e_i) = \begin{cases} 2i-1 & \text{if } 1 \leq i \leq r+1 \\ 2i & \text{if } r+2 \leq i \leq n \end{cases}$$

It is easy to observe that f is injective. The induced vertex labels are given as follows:

$$f^v(v_i) = \begin{cases} n+1 & \text{if } i = 1 \\ 2i-2 & \text{if } 2 \leq i \leq r+1 \\ 2i-1 & \text{if } r+2 \leq i \leq n \end{cases}$$

It is clear that,

$$f(E) \cup f^v(V) = \{1, 2, 3, \cdots, 2n\}$$

Case 2: $n \equiv 0(mod\ 2)$.

Let C_n be an even cycle with n vertices. Let $\{e_1, e_2, \cdots, e_n\}$ be the edge set and $\{v_1, v_2, \cdots, v_n\}$ be the vertex set of C_n such that $e_i = v_i v_{i+1}, 1 \leq i \leq n-1$ and $e_n = v_n v_1$.

Let $n = 2r$. The edges of C_n are labeled as follows:

$$f(e_i) = \begin{cases} 1 & \text{if } i = 1 \\ 3 & \text{if } i = 2 \\ 7 & \text{if } i = 3 \\ 4i - 4 & \text{if } 4 \leq i \leq r+1 \\ 4n - 4i + 5 & \text{if } r+2 \leq i \leq n-1 \\ 6 & \text{if } i = n \end{cases}$$

It is easy to observe that f is injective. The induced vertex labels are given as follows:

$$f(e_i) = \begin{cases} 4 & \text{if } i = 1 \\ 2 & \text{if } i = 2 \\ 5 & \text{if } i = 3 \\ 4i - 6 & \text{if } 4 \leq i \leq r+1 \\ 4n - 4i + 7 & \text{if } r+2 \leq i \leq n-1 \\ 8 & \text{if } i = n \end{cases}$$

It is clear that,

$$f(E) \cup f^v(V) = \{1, 2, 3, \cdots, 2n\}$$

Hence we know that all Cycles C_n, except C_4 are Super Vertex Mean graphs.

3.2 Result 2: All Odd Cycles can be SVM Labeled as many as $\lfloor \frac{n}{2} \rfloor$ Different Ways and Every Even Cycle, Except C_4 can have $\left(\lfloor \frac{n}{2} \rfloor - 1\right)$ Types of SVM Labelings [8]

Let $n \equiv 1(mod\ 2)$, and $n = 2r + 1$. Let C_n be an odd cycle with n vertices. Let $\{e_1, e_2, \cdots, e_n\}$ be the edge set and $\{v_1, v_2, \cdots, v_n\}$ be the vertex set of C_n such that $e_i = v_i v_{i+1}, 1 \leq i \leq n-1$ and $e_n = v_n v_1$. Then the s-type $(1 \leq s \leq r)$ SVM labeling of cycle $C_n, n \equiv 1(mod\ 2), n \geq 3$ is given as follows:

$$f_s(e_i) = \begin{cases} 1 & \text{if } i = 1 \\ 4i - 2 & \text{if } 2 \leq i \leq s \\ 4r + 2s - 2i + 4 & \text{if } s + 1 \leq i \leq r + 1 \\ 4r + 2s - 2i + 3 & \text{if } r + 2 \leq i \leq 2r - s + 2 \\ 8r - 4i + 7 & \text{if } 2r - s + 3 \leq i \leq n. \end{cases}$$

Clearly f_s is an injective function with range from $\{1, 2, \cdots, 2n\}$.

The induced vertex labeling is given as follows:

$$f_s^v(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 4i - 4 & \text{if } 2 \leq i \leq s \\ 2r + 2s & \text{if } i = s + 1 \\ 4r + 2s - 2i + 5 & \text{if } s + 2 \leq i \leq r + 2 \\ 4r + 2s - 2i + 4 & \text{if } r + 3 \leq i \leq 2r - s + 2 \\ 8r - 4i + 9 & \text{if } 2r - s + 3 \leq i \leq n. \end{cases}$$

Clearly it is injective and

$$f_s(E) \cup f_s^v(V) = \{1, 2, 3, 4, 5, \cdots, 2n - 3, 2n - 2, 2n - 1, 2n\}.$$

Let $n \equiv 0(mod\ 2)$. Let C_n be an even cycle and $n = 2r$ where, $n \geq 6$. (Since C_4 is not an SVM graph). Let $\{e_1, e_2, \cdots, e_n\}$ be the edge set and $\{v_1, v_2, \cdots, v_n\}$ be the vertex set of C_n such that $e_i = v_i v_{i+1}, 1 \leq i \leq n - 1$ and $e_n = v_n v_1$. Checking various possibilities we realize that Type 1 labeling is not possible for even cycles. So we assume that $2 \leq s \leq r$.

The s-type $(2 \leq s \leq r - 1)$ SVM labeling of C_n, $n \equiv 0(mod\ 2), n \geq 6$, is given as follows:

$$f_s(e_i) = \begin{cases} 1 & \text{if } i = 1 \\ 7 & \text{if } i = 2 \\ 4i & \text{if } 3 \leq i \leq s \\ 4r - 2i + 2s + 2 & \text{if } s + 1 \leq i \leq r \\ 4r - 2i + 2s + 1 & \text{if } r + 1 \leq i \leq 2r - s \\ 8r - 4i + 1 & \text{if } 2r - s + 1 \leq i \leq 2r - 2 \\ 6r - 3i + 3 & \text{if } 2r - 1 \leq i \leq n \end{cases}$$

and, when $s = r$, the $r-$type labeling is given by,

$$f_r(e_i) = \begin{cases} 1 & \text{if } i = 1 \\ 4i - 2 & \text{if } 2 \leq i \leq r - 1 \\ r + 3i - 3 & \text{if } r \leq i \leq r + 1 \\ 8r - 4i + 3 & \text{if } r + 2 \leq i \leq n. \end{cases}$$

Clearly f_s is an injective function with range from $\{1, 2, ..., 2n\}$.

The induced vertex labeling is given as follows:

When $s = 2$

$$f_s^v(v_i) = \begin{cases} 2i & \text{if } 1 \leq i \leq 2 \\ 2r + 4 & \text{if } i = 3 \\ 4r - 2i + 2s + 3 & \text{if } 4 \leq i \leq r + 1 \\ 4r - 2i + 2s + 2 & \text{if } r + 2 \leq i \leq 2r - 2 \\ 6r - 3i + 5 & \text{if } 2r - 1 \leq i \leq n \end{cases}$$

And when $3 \leq s \leq r - 1$, we have

$$f_s^v(v_i) = \begin{cases} 2i & \text{if } 1 \leq i \leq 2 \\ 2i + 4 & \text{if } i = 3 \\ 4i - 2 & \text{if } 4 \leq i \leq s \\ 2r + 2s & \text{if } i = s + 1 \\ 4r - 2i + 2s + 3 & \text{if } s + 2 \leq i \leq r + 1 \\ 4r - 2i + 2s + 2 & \text{if } r + 2 \leq i \leq 2r - s \\ 4s - 1 & \text{if } i = 2r - s + 1 \\ 8r - 4i + 3 & \text{if } 2r - s + 2 \leq i \leq 2r - 2 \\ 6r - 3i + 5 & \text{if } 2r - 1 \leq i \leq n \end{cases}$$

Clearly it is an injective function and, it is also evident that, when $s = 2$,

$$f_2(E) \cup f_2^v(V) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \cdots, 4r - 3, 4r - 2, 4r - 1, 4r\}.$$

Also for $3 \leq s \leq r - 1$,

$$f_s(E) \cup f_s^v(V) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots, 4r - 2, 4r - 1, 4r\}.$$

When $s = r$, the induced vertex labeling is given as follows:

$$f_r^v(v_i) = \begin{cases} 2i & \text{if } 1 \leq i \leq 2 \\ 4i - 4 & \text{if } 3 \leq i \leq r \\ 4r - 1 & \text{if } i = r + 1 \\ 4r - 2 & \text{if } i = r + 2 \\ 8r - 4i + 5 & \text{if } r + 3 \leq i \leq n. \end{cases}$$

so that,

$$f_r(E) \cup f_r^v(V) = \{1, 2, 3, \cdots, 2n - 2, 2n - 1, 2n\}.$$

3.3 Result 3: A Triangular Snake with k Blocks is an SVM Graph [7]

Let kC_3 be a triangular snake with k blocks with p vertices and q edges. Then $p = 2k + 1$ and $q = 3k$. Let $V(kC_3) = \{u_i : 1 \leq i \leq k + 1\} \cup \{v_i : 1 \leq i \leq k\}$ and $E(kC_3) = \{u_i u_{i+1}, u_i v_i, u_{i+1} v_i : 1 \leq i \leq k\}$.
The edges of kC_3 are labeled as follows:

$$f(u_i u_{i+1}) = \begin{cases} 1 & \text{if } i = 1 \\ 5i & \text{if } i \text{ is even and } i \neq k \\ 5i - 3 & \text{if } i \text{ is odd and } i \neq 1 \\ 5k + 1 & \text{if } n \text{ is even and } i = k \end{cases}$$

$$f(u_i v_i) = \begin{cases} 5i - 3 & \text{if } i \text{ is even} \\ 5i - 2 & \text{if } i \text{ is odd} \end{cases}$$

$$f(v_i v_{i+1}) = \begin{cases} 5i - 1 & \text{if } i \text{ is even} \\ 5i & \text{if } i \text{ is odd and } i \neq k \\ 5k + 1 & \text{if } k \text{ is odd and } i = k \end{cases}$$

Then, the induced vertex labels are as follows:

$$f^v(u_i) = \begin{cases} 2 & \text{if } i = 1 \\ 5i - 4 & \text{if } 2 \leq i \leq k \\ 5k & \text{if } i = k + 1 \text{ and k is even} \\ 5i - 1 & \text{if } i = k + 1 \text{ and k is odd} \end{cases}$$

$$f^v(v_i) = \begin{cases} 5i - 2 & \text{if } i \text{ is even} \\ 5i - 1 & \text{if } i \text{ is odd and } i \neq k \\ 5k & \text{if } i = k \text{ is odd} \end{cases}$$

It can be easily verified that f is injective and the set of edge labels and induced vertex labels is $\{1, 2, ..., 5k + 1\}$.

Example 3.1 *Super vertex mean labeling of triangular snakes is shown in Figure* 1.

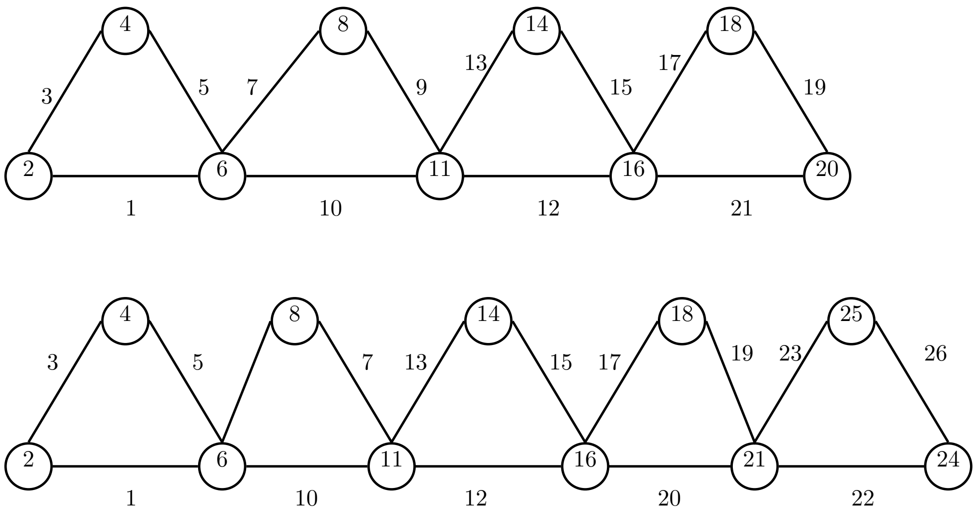

Figure 1: Super vertex mean labeling of triangular snakes

3.4 Result 4: Quadrilateral Snakes with $n \geq 2$ Blocks and Each $s_i = 1$ are SVM Graphs [7]

Let kC_4 be a quadrilateral snake with $V(kC_4) = \{u_i : 1 \leq i \leq k+1\} \cup \{u_i, w_i : 1 \leq i \leq k\}$ and $E(kC_4) = \{u_iu_{i+1}, u_iv_i, u_{i+1}w_i, v_iw_i : 1 \leq i \leq k\}$.

Then $p = 3k + 1$ and $q = 4k$.

Define $f : E(kC_4) \to \{1, 2, 3, ..., 7k + 1\}$ as follows:

$$f(u_iu_{i+1}) = \begin{cases} 7i & \text{if } 1 \leq i \leq k-1 \\ 7k+1 & \text{if } i = k. \end{cases}$$

$$f(u_iv_i) = 7i - 6 \text{ if } 1 \ \leq i \leq k.$$

$$f(v_iw_i) = \begin{cases} 3 & \text{if } i = k \\ 7i-3 & \text{if } 2 \leq i \leq k. \end{cases}$$

$$f(w_iu_{i+1}) = 7i - 1 \text{ if } 1 \ \leq i \leq k.$$

Then, the induced vertex labels are as follows:

$$f^v(u_i) = \begin{cases} 4 & \text{if } i = 1 \\ 7k & \text{if } i = k+1 \\ 7i-5 & \text{otherwise.} \end{cases}$$

$$f^v(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 7i-5 & \text{otherwise.} \end{cases}$$

$$f^v(w_i) = 7i - 2 \text{ if } 1 \leq i \leq k.$$

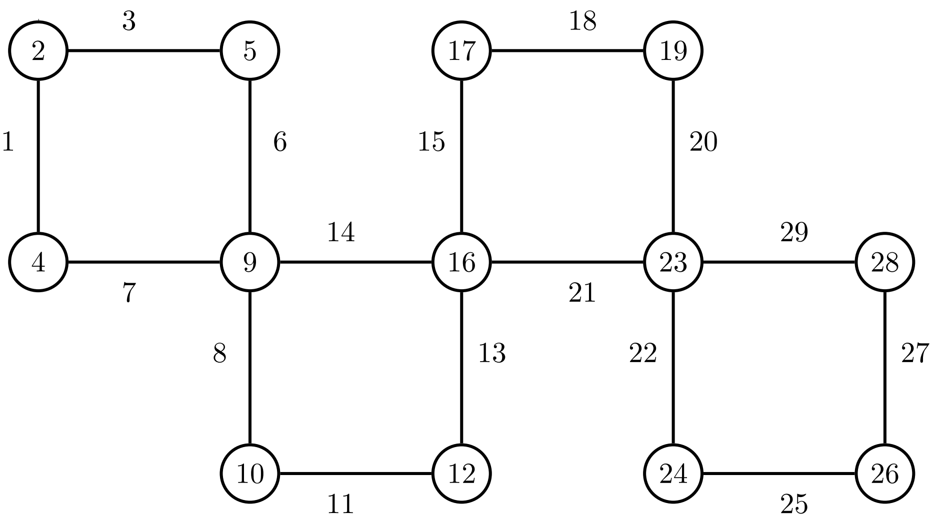

Figure 2: A Super vertex-mean labeling of a Quadrilateral snake

It can be easily verified that f is injective and the set of edge labels and induced vertex labels is $\{1, 2, 3, ..., 7k + 1\}$.

Example 3.2 *A Super vertex-mean labeling of a Quadrilateral snake is shown in Figure* 2*.*

4 Cyclic Snakes Made of Cycles of Higher Orders

Now we proceed to prove that cyclic snakes of cycles of other orders are also SVM graphs.

Theorem 4.1 *Pentagonal snakes with k blocks and each $s_i = 1$ are SVM graphs.*

Proof:
Let kC_5 be a pentagonal snake with k blocks of C_5.

Let $V(kC_5) = \{v_{i,j}; 1 \le i \le k, 1 \le j \le 5\}$ and

$E(kC_5) = \{e_{i,j} = v_{i,j}v_{i,j+1}$ and $e_{i,5} = v_{i,5}v_{i,1}; 1 \le i \le k, 1 \le j \le 4\}$.

Note that $v_{i,5} = v_{i+1,1}$ for $1 \le i \le k-1$, and we refer this vertex as $v_{i,5}$ throughout this proof.

Now, $p = 4k + 1$, $q = 5k$ and $p + q = 9k + 1$.

Define $f : E(kC_5) \to \{1, 2, 3, \cdots, 9k + 1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 2j-1, & \text{if } i=1, \text{ and } 1 \le j \le 3 \\ 2j, & \text{if } i=1, \text{ and } 4 \le j \le 5 \\ 9i-9, & \text{if } 2 \le i \le k \text{ and } j=1 \\ 9i+2j-9, & \text{if } 2 \le i \le k \text{ and } 2 \le j \le 5. \end{cases}$$

It can be easily verified that f is injective.

Then, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} n+1, & \text{if } i=1, \text{ and } j=1 \\ 2j-2, & \text{if } i=1, \text{ and } 2 \le j \le 3 \\ 2j-1, & \text{if } i=1, \text{ and } j=4 \\ 9i+3, & \text{if } 1 \le i \le k-1 \text{ and } j=5 \\ 9i-7, & \text{if } 2 \le i \le k \text{ and } j=2 \\ 9i+2j-10, & \text{if } 2 \le i \le k, \text{ and } 3 \le j \le 5 \\ 9k, & \text{if } i=k, \text{ and } j=5. \end{cases}$$

Clearly it can be proved that the union of the set of edge labels and the induced vertex labels is $\{1, 2, 3, \cdots, 9k + 1\}$.

Therefore, pentagonal snakes kC_5 with each $s_i = 1$ are Super Vertex Mean graphs.

Example 4.2 *Given in Figure* 3*, is an SVM labeling of a pentagonal snake with* 4 *blocks.*

Theorem 4.3 *Hexagonal snakes with each* $s_i = 1, (1 \le i \le k-2)$ *are Super Vertex Mean Graphs.*

Proof: Let kC_6 be a hexagonal snake with k blocks of C_6.

Let $V(kC_6) = \{v_{i,j}; 1 \le i \le k, 1 \le j \le 6\}$ and

$E(kC_6) = \{e_{i,j} = v_{i,j}v_{i,j+1}$ and $e_{i,6} = v_{i,6}v_{i,1}; 1 \le i \le k, 1 \le j \le 5\}$.

Note that $v_{i,6} = v_{i+1,1}$ for $1 \le i \le k-1$, and we refer this vertex as $v_{i,6}$ throughout this proof.

Now, $p = 5k + 1$ and $q = 6k$ and $p + q = 11k + 1$.

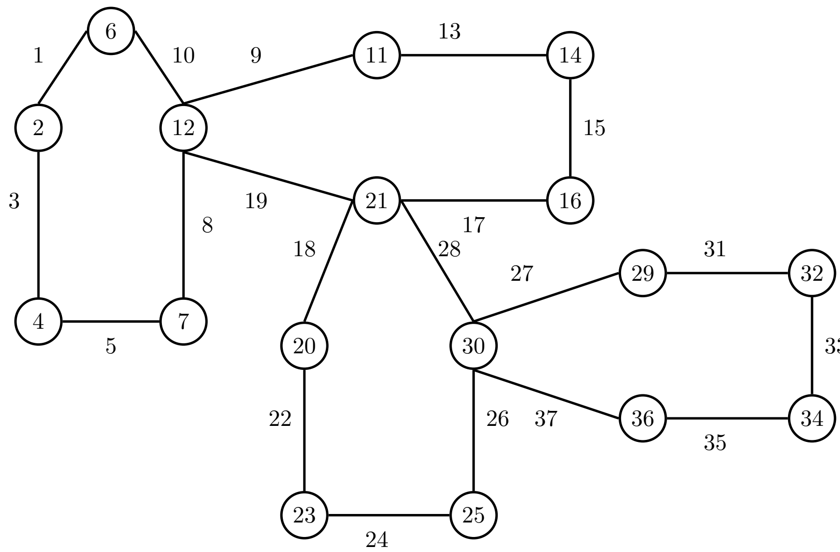

Figure 3: A Super vertex-mean labeling of a pentagonal snake with 4 blocks

Define $f : E(G_n) \rightarrow \{1, 2, 3, \cdots, 11k + 1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 9 - 3j, & \text{if } i = 1, \text{ and } 1 \leq j \leq 2 \\ 6j - 17, & \text{if } i = 1, \text{ and } 3 \leq j \leq 4 \\ 27 - 3j, & \text{if } i = 1, \text{ and } 5 \leq j \leq 6 \\ 11i - 11, & \text{if } 2 \leq i \leq k, \text{ and } j = 1 \\ 11i + 2j - 11, & \text{if } 2 \leq i \leq k, \text{ and } 2 \leq j \leq 6. \end{cases}$$

It can be easily verified that f is injective.

Then, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} 11 - 3j, & \text{if } i = 1, \text{ and } 1 \leq j \leq 3 \\ 6j - 20, & \text{if } i = 1, \text{ and } 4 \leq j \leq 5 \\ 11i + 3, & \text{if } 1 \leq i \leq k - 1, \text{ and } j = 6 \\ 11i - 9, & \text{if } 2 \leq i \leq k, \text{ and } j = 2 \\ 11i + 2j - 12, & \text{if } 2 \leq i \leq k, \text{ and } 3 \leq j \leq 5 \\ 11k, & \text{if } i = k, \text{ and } j = 6. \end{cases}$$

Clearly it can be proved that the union of the set of edge labels and the induced vertex labels is $\{1, 2, 3, \cdots, 11k + 1\}$.

Therefore, hexagonal snakes with k blocks of C_6 are Super Vertex Mean graphs.

Example 4.4 *Given in Figure* 4 *is an SVM labeling of a hexagonal snake with* 3 *blocks.*

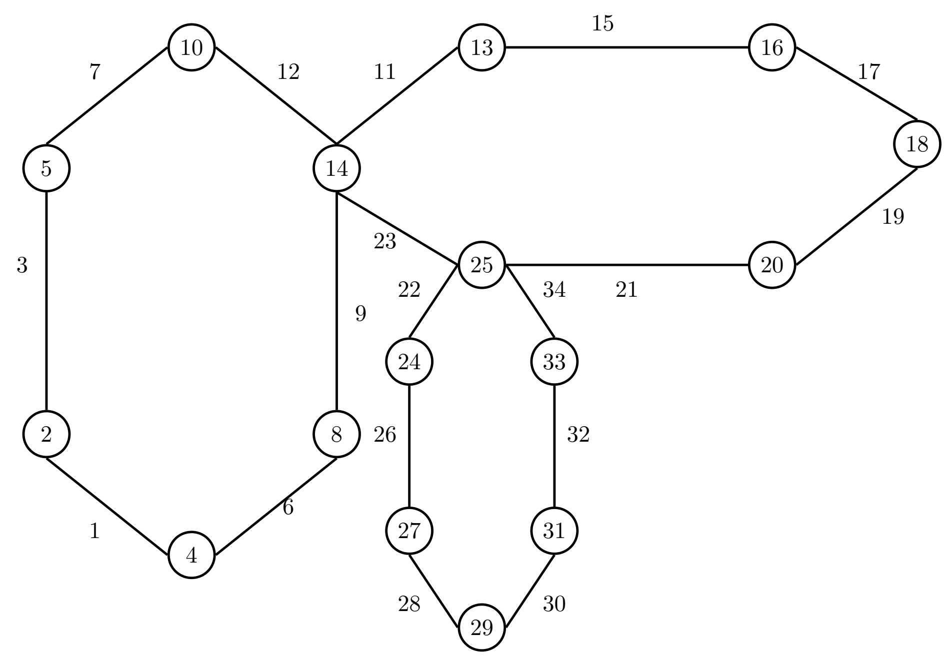

Figure 4: An SVM labeling of a hexagonal snake with 3 blocks

Theorem 4.5 *Let kC_n be a cyclic snake with k blocks of $C_n, n \geq 7$ and $n \equiv 3(mod\ 4)$. Then kC_n is a Super Vertex Mean graph.*

Proof: Let kC_n be a cyclic snake with k blocks of $C_n, n \geq 7$ and $n \equiv 3(mod\ 4)$.

Let $n = 2r + 1$, and $r = 2s + 1$ so that $n = 4s + 3$.

Let $V(kC_n) = \{v_{i,j}; 1 \leq i \leq k, 1 \leq j \leq n\}$ and $E(kC_n) = \{e_{i,j} = v_{i,j}v_{i,j+1}$ & $e_{i,n} = v_{i,n}v_{i,1}; 1 \leq i \leq k, 1 \leq j \leq n - 1\}$.

Note that $v_{i,n} = v_{i+1,1}$ for $1 \leq i \leq k - 1$, and we refer this vertex as $v_{i,n}$ throughout this proof.

Now, $p = (n - 1)k + 1$ and $q = nk$ and $p + q = (2n - 1)k + 1$.

Define $f : E(kC_n) \to \{1, 2, 3, \cdots, (2n-1)k+1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 2j-1, & \text{if } i=1, \text{ and } 1 \le j \le r+1 \\ 2j, & \text{if } i=1, \text{ and } r+2 \le j \le n \\ (2n-1)i-(2n-1), & \text{if } 2 \le i \le k \text{ and } j=1 \\ (2n-1)i+2j-(2n), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le r-s \\ (2n-1)i+2j-(2n-1), & \text{if } 2 \le i \le k \text{ and } r-s+1 \le j \le n. \end{cases}$$

$$= \begin{cases} 2j-1, & \text{if } i=1, \text{ and } 1 \le j \le 2s+2 \\ 2j, & \text{if } i=1, \text{ and } 2s+3 \le j \le 4s+3 \\ (8s+5)i-(8s+5), & \text{if } 2 \le i \le k \text{ and } j=1 \\ (8s+5)i+2j-(8s+6), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le s+1 \\ (8s+5)i+2j-(8s+5), & \text{if } 2 \le i \le k \text{ and } s+2 \le j \le 4s+3. \end{cases}$$

And, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} n+1, & \text{if } i=1, \text{ and } j=1 \\ 2j-2, & \text{if } i=1, \text{ and } 2 \le j \le r+1 \\ 2j-1, & \text{if } i=1 \text{ and } r+2 \le j \le n-1 \\ (2n-1)i+r+1, & \text{if } 1 \le i \le k-1 \text{ and } j=n \\ (2n-1)i+2j-(2n+1), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le r-s \\ (2n-1)i+2j-(2n), & \text{if } 2 \le i \le k \text{ and } r-s+1 \le j \le n-1 \\ (2n-1)k & \text{if } i=k \text{ and } j=n. \end{cases}$$

$$= \begin{cases} 4s+4, & \text{if } i=1, \text{ and } j=1 \\ 2j-2, & \text{if } i=1, \text{ and } 2 \le j \le 2s+2 \\ 2j-1, & \text{if } i=1 \text{ and } 2s+3 \le j \le 4s+2 \\ (8s+5)i+2s+2, & \text{if } 1 \le i \le k-1 \text{ and } j=4s+3 \\ (8s+5)i+2j-(8s+7), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le s+1 \\ (8s+5)i+2j-(8s+6), & \text{if } 2 \le i \le k \text{ and } s+2 \le j \le 4s+2 \\ (8s+5)k & \text{if } i=k \text{ and } j=4s+3. \end{cases}$$

We prove the theorem by using mathematical induction on s.

When $s = 1, r = 3$ and $n = 7$ and the cyclic snake is a heptagonal snake with k cycles of C_7 .

Now, $p = 7k + 1$ and $q = 7k$ and $p + q = 13k + 1$.

Define $f : E(kC_n) \to \{1, 2, 3, \cdots, 13k + 1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 2j-1, & \text{if } i=1, \text{ and } 1 \le j \le 4 \\ 2j, & \text{if } i=1, \text{ and } 5 \le j \le 7 \\ 13i-13, & \text{if } 2 \le i \le k, \text{ and } j=1 \\ 13i+2j-14, & \text{if } 2 \le i \le k, \text{ and } j=2 \\ 13i+2j-13, & \text{if } 2 \le i \le k, \text{ and } 3 \le j \le 7. \end{cases}$$

It can be easily verified that f is injective.

Then, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} n+1, & \text{if } i=1, \text{ and } j=1 \\ 2j-2, & \text{if } i=1, \text{ and } 2 \le j \le 4 \\ 2j-1, & \text{if } i=1, \text{ and } 5 \le j \le 6 \\ 13i+14, & \text{if } 1 \le i \le k-1, \text{ and } j=7 \\ 13i-11, & \text{if } 2 \le i \le k, \text{ and } j=2 \\ 13i+2j-14, & \text{if } 2 \le i \le k \text{ and } 3 \le j \le 6 \\ 13k, & \text{if } i=k, \text{ and } j=7. \end{cases}$$

Clearly it can be proved that the union of the set of edge labels and the induced vertex labels is $\{1, 2, 3, \cdots, 13k + 1\}$.

Let,

$$\begin{aligned} A_1 &= \{2j-1, i=1 \ \& \ 1 \le j \le 4\}, \\ A_2 &= \{2j, i=1 \ \& \ 5 \le j \le 7\}, \\ A_3 &= \{13i-13, 2 \le i \le k \ \& \ j=1\}, \\ A_4 &= \{13i+2j-14, 2 \le i \le k \ \& \ j=2\}, \\ A_5 &= \{13i+2j-13, 2 \le i \le k \ \& \ 3 \le j \le 7\}. \end{aligned}$$

And let,

$$\begin{aligned} B_1 &= \{8\}, \\ B_2 &= \{2, 4, 6\}, \\ B_3 &= \{9, 11\}, \\ B_4 &= \{17, 30, 43, 56, \cdots, 13k-22, 13k-9\}, \\ B_5 &= \{15, 28, 41, \cdots, 13k-24, 13k-11\}, \\ B_6 &= \{18, 20, 22, 24, \cdots, 13k-8, 13k-6, 13k-4, 13k-2\}, \\ B_7 &= \{13k\}. \end{aligned}$$

$$
\begin{aligned}
A_1 \cup B_2 \cup B_1 \cup B_3 \cup A_2 &= \{1, 2, 3, 4 \cdots, 11, 12, 14\}, \\
A_3 &= \{13, 26, 39, \cdots, 13k - 13\}, \\
B_5 \cup A_4 \cup B_4 \cup B_6 \cup A_5 &= \{15, 16, \cdots, 24, 25, 27, 28, \cdots, \\
&\quad 38, 40, \cdots, 13k - 1, 13k + 1\}, \\
A_1 \cup B_2 \cup B_1 \cup B_3 \cup A_2 \cup A_3 \cup B_5 \cup A_4 \cup & \\
& B_4 \cup B_6 \cup A_5 \cup B_7 \\
&= \{1, 2, 3, \cdots, 13k - 1, 13k, 13k + 1\}.
\end{aligned}
$$

Thus the theorem is true when $s = 1$.
Now we assume that the theorem is true for some $s - 1$ (i.e., for $r - 2$ and $n - 4$).

The induction hypothesis is that the edge labeling,

$$f : E(kC_{n-4}) \to \{1, 2, 3, \cdots, (2n - 9)k + 1\},$$

defined as follows, is a Super Vertex Mean Labeling, where $n \geq 11$ and $n \equiv 3(mod\, 4)$ and $k \geq 2$.

$$
f(e_{i,j}) = \begin{cases}
2j - 1, & \text{if } i = 1, \text{ and } 1 \leq j \leq r - 1 \\
2j, & \text{if } i = 1, \text{ and } r \leq j \leq n - 4 \\
(2n - 9)i - (2n - 9), & \text{if } 2 \leq i \leq k \text{ and } j = 1 \\
(2n - 9)i + 2j - (2n - 8), & \text{if } 2 \leq i \leq k \text{ and } 2 \leq j \leq r - s - 1 \\
(2n - 9)i + 2j - (2n - 9), & \text{if } 2 \leq i \leq k \text{ and } r - s \leq j \leq n - 4.
\end{cases}
$$

$$
= \begin{cases}
2j - 1, & \text{if } i = 1, \text{ and } 1 \leq j \leq 2s \\
2j, & \text{if } i = 1, \text{ and } 2s + 1 \leq j \leq 4s - 1 \\
(8s - 3)i - (8s - 3), & \text{if } 2 \leq i \leq k \text{ and } j = 1 \\
(8s - 3)i + 2j - (8s - 2), & \text{if } 2 \leq i \leq k \text{ and } 2 \leq j \leq s \\
(8s - 3)i + 2j - (8s - 3), & \text{if } 2 \leq i \leq k \text{ and } s + 1 \leq j \leq 4s - 1.
\end{cases}
$$

Now we prove that the result is true for any s. If we replace s with $s + 1$ in the above mappings we get,

$$f(e_{i,j}) = \begin{cases} 2j-1, & \text{if } i=1, \text{ and } 1 \le j \le 2s+2 \\ 2j, & \text{if } i=1, \text{ and } 2s+3 \le j \le 4s+3 \\ (8s+5)i-(8s+5), & \text{if } 2 \le i \le k \text{ and } j=1 \\ (8s+5)i+2j-(8s+6), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le s+1 \\ (8s+5)i+2j-(8s+5), & \text{if } 2 \le i \le k \text{ and } s+2 \le j \le 4s+3. \end{cases}$$

$$= \begin{cases} 2j-1, & \text{if } i=1, \text{ and } 1 \le j \le r+1 \\ 2j, & \text{if } i=1, \text{ and } r+2 \le j \le n \\ (2n-1)i-(2n-1), & \text{if } 2 \le i \le k \text{ and } j=1 \\ (2n-1)i+2j-(2n), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le r-s \\ (2n-1)i+2j-(2n-1), & \text{if } 2 \le i \le k \text{ and } r-s+1 \le j \le n. \end{cases}$$

And, the induced vertex label is,

$$f^v(v_{i,j}) = \begin{cases} 4s+4, & \text{if } i=1, \text{ and } j=1 \\ 2j-2, & \text{if } i=1, \text{ and } 2 \le j \le 2s+2 \\ 2j-1, & \text{if } i=1 \text{ and } 2s+3 \le j \le 4s+2 \\ (8s+5)i+2s+2, & \text{if } 1 \le i \le k-1 \text{ and } j=4s+3 \\ (8s+5)i+2j-(8s+7), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le s+1 \\ (8s+5)i+2j-(8s+6), & \text{if } 2 \le i \le k \text{ and } s+2 \le j \le 4s+2 \\ (8s+5)k & \text{if } i=k \text{ and } j=4s+3. \end{cases}$$

$$= \begin{cases} n+1, & \text{if } i=1, \text{ and } j=1 \\ 2j-2, & \text{if } i=1, \text{ and } 2 \le j \le r+1 \\ 2j-1, & \text{if } i=1 \text{ and } r+2 \le j \le n-1 \\ (2n-1)i+r+1, & \text{if } 1 \le i \le k-1 \text{ and } j=n \\ (2n-1)i+2j-(2n+1), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le r-s \\ (2n-1)i+2j-(2n), & \text{if } 2 \le i \le k \text{ and } r-s+1 \le j \le n-1 \\ (2n-1)k & \text{if } i=k \text{ and } j=n. \end{cases}$$

It is clear that $f(E) \cup f^v(V) = \{1, 2, 3, \cdots, (2n)k, (2n-1)k, (2n-1)k+1\}$
Thus the theorem is proved by Mathematical Induction.

Example 4.6 *SVM Labeling of Undecagonal snake with* 4 *blocks of* C_{11} *is given in Figure* 5.

Theorem 4.7 *Let* kC_n *be a cyclic snake with* k *blocks of* $C_n, n \ge 8$ *and* $n \equiv 0(mod\ 4)$. *Then* kC_n *is a Super Vertex Mean graph.*

Proof: Let kC_n be a cyclic snake with k blocks of $C_n, n \ge 8$ and $n \equiv 0(mod\ 4)$.

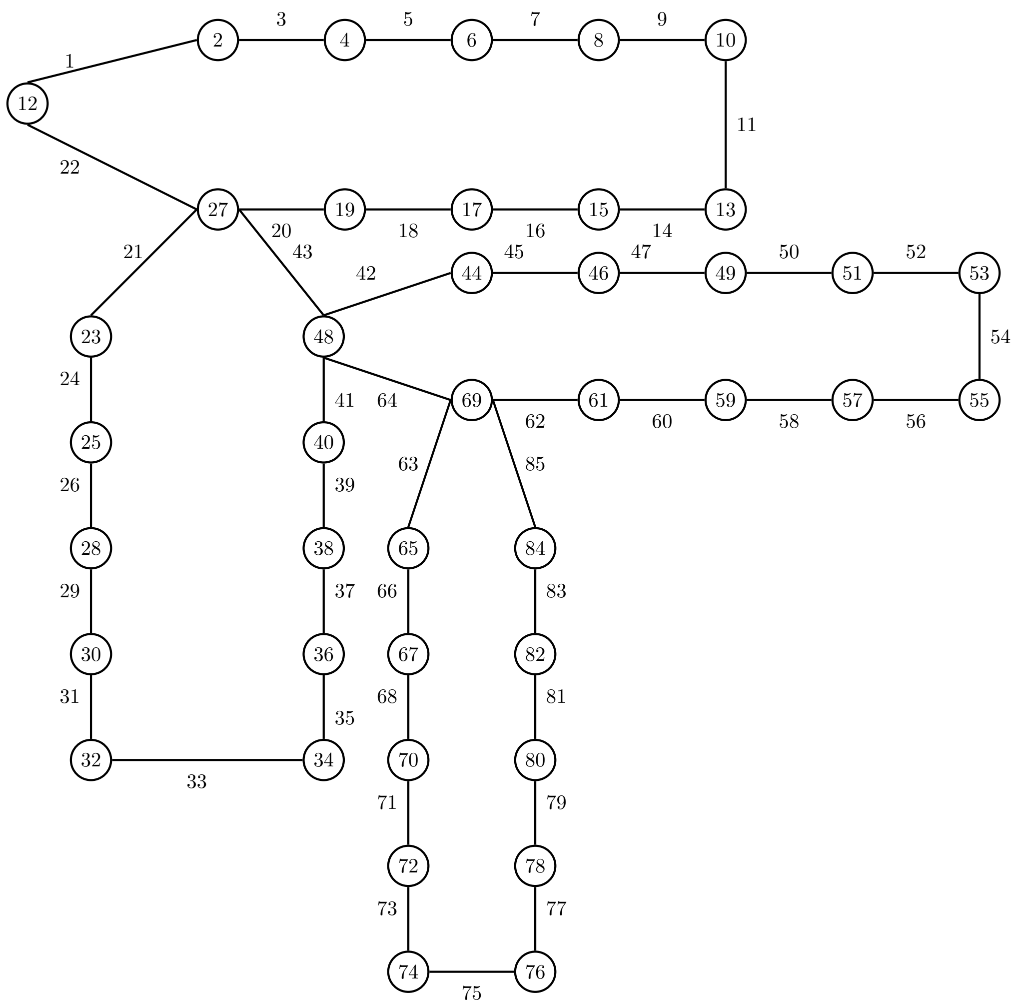

Figure 5: An SVM labeling of an Undecagonal (C_{11}) – Snake with 4 blocks

Let $n = 2r$, and $r = 2s$ so that $n = 4s$.

Let $V(kC_n) = \{v_{i,j}; 1 \le i \le k, 1 \le j \le n\}$ and $E(kC_n) = \{e_{i,j} = v_{i,j}v_{i,j+1}$ & $e_{i,n} = v_{i,n}v_{i,1}; 1 \le i \le k, 1 \le j \le n-1\}$.

Note that $v_{i,n} = v_{i+1,1}$ for $1 \le i \le k-1$, and we refer this vertex as $v_{i,n}$ throughout this proof.

Now, $p = (n-1)k + 1$ and $q = nk$ and $p + q = (2n-1)k + 1$.

Define $f : E(kC_n) \to \{1, 2, 3, \cdots, (2n-1)k + 1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 2n-2j-2, & \text{if } i = 1, \text{ and } 1 \le j \le r-4 \\ 2n-2j-3, & \text{if } i = 1, \text{ and } r-3 \le j \le n-6 \\ 3n-3j-3, & \text{if } i = 1, \text{ and } n-5 \le j \le n-4 \\ 1, & \text{if } i = 1, \text{ and } j = n-3 \\ 7, & \text{if } i = 1, \text{ and } j = n-2 \\ 4n-2j-2, & \text{if } i = 1, \text{ and } n-1 \le j \le n \\ (2n-1)i-(2n-1), & \text{if } 2 \le i \le k \text{ and } j = 1 \\ (2n-1)i+2j-2n, & \text{if } 2 \le i \le k \text{ and } 2 \le j \le s \\ (2n-1)i+2j-(2n-1), & \text{if } 2 \le i \le k \text{ and } s+1 \le j \le n. \end{cases}$$

And, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} 2n-2j-1, & \text{if } i = 1, \text{ and } 1 \le j \le r-3 \\ 2n-2j-2, & \text{if } i = 1, \text{ and } r-2 \le j \le n-5 \\ 5, & \text{if } i = 1, \text{ and } j = n-4 \\ 8+2j-2n, & \text{if } i = 1, \text{ and } n-3 \le j \le n-2 \\ n+4, & \text{if } i = 1, \text{ and } j = n-1 \\ (2n-1)i+r, & \text{if } 1 \le i \le k-1 \text{ and } j = n \\ (2n-1)i+2j-(2n+1), & \text{if } 2 \le i \le k \text{ and } 2 \le j \le s \\ (2n-1)i+2j-2n, & \text{if } 2 \le i \le k \text{ and } s+1 \le j \le n-1 \\ (2n-1)k, & \text{if } i = k \text{ and } j = n. \end{cases}$$

It can be easily proved using mathematical induction on s as in the above theorem that the labeling $f : E(kC_n) \to \{1, 2, 3, \ldots, (2n-1)k + 1\}$ is an SVM labeling.

Hint: Wherever r and n appear, we need to change those variables into s using $n = 4s$ and $r = 2s$.

Example 4.8 *A Dodeagonal* (C_{12}) *snake with* 4 *blocks is an SVM graph as shown in Figure* 6*.*

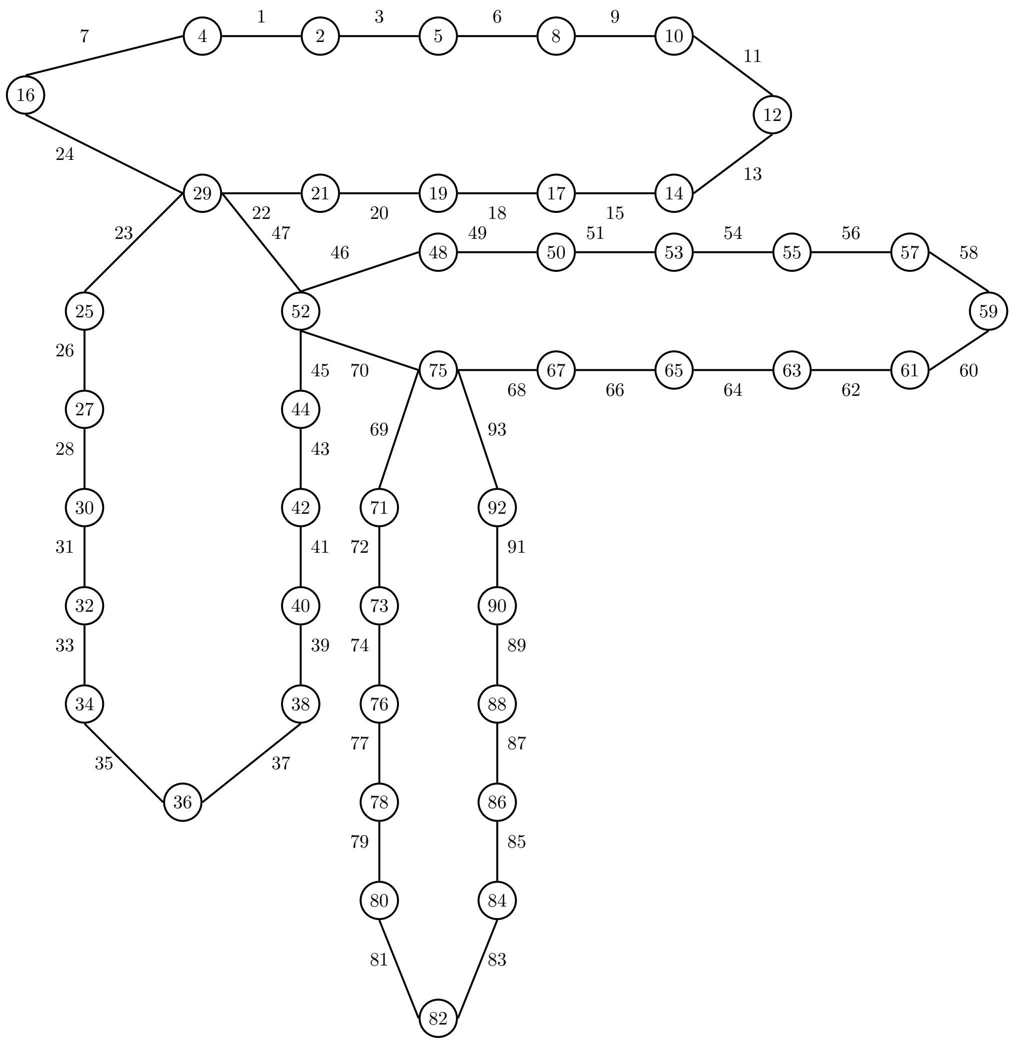

Figure 6: An SVM labeling of a Dodecagonal (C_{12}) snake with 4 blocks.

Theorem 4.9 *Let kC_n be a cyclic snake with k blocks of $C_n, n \geq 9$ and $n \equiv 1(mod\ 4)$. Then kC_n is a Super Vertex Mean graph.*

Proof: Let kC_n be a cyclic snake with k blocks of $C_n, n \geq 9$ and $n \equiv 1(mod\ 4)$.

Let $n = 2r + 1$, and $r = 2s$ so that $n = 4s + 1$.

Let $V(kC_n) = \{v_{i,j}; 1 \leq i \leq k, 1 \leq j \leq n\}$ and $E(kC_n) = \{e_{i,j} = v_{i,j}v_{i,j+1}$ & $e_{i,n} = v_{i,n}v_{i,1}; 1 \leq i \leq k, 1 \leq j \leq n-1\}$.

Note that $v_{i,n} = v_{i+1,1}$ for $1 \leq i \leq k-1$, and we refer this vertex as $v_{i,n}$ throughout this proof.

Now, $p = (n-1)k + 1$ and $q = nk$ and $p + q = (2n-1)k + 1$.

Define $f : E(kC_n) \to \{1, 2, 3, \cdots, (2n-1)k + 1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 2j-1, & \text{if } i = 1, \text{ and } 1 \leq j \leq r+1 \\ 2j, & \text{if } i = 1, \text{ and } r+2 \leq j \leq n \\ (2n-1)i - 2j - 8, & \text{if } 2 \leq i \leq k \text{ and } 1 \leq j \leq r-3 \\ (2n-1)i - 2j - 6, & \text{if } 2 \leq i \leq k \text{ and } r-2 \leq j \leq n-7 \\ (2n-1)i - 2n + 5, & \text{if } 2 \leq i \leq k \text{ and } j = n-6, \\ (2n-1)i - 2n + 2j + 1, & \text{if } 2 \leq i \leq k \text{ and } n-5 \leq j \leq n. \end{cases}$$

And, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} n+1, & \text{if } i = 1, \text{ and} j = 1 \\ 2j-2, & \text{if } i = 1, \text{ and } 2 \leq j \leq r+1 \\ 2j-1, & \text{if } i = 1, \text{ and } r+2 \leq j \leq n-1 \\ 3n+4, & \text{if } 1 \leq i \leq k-1 \text{ and } j = n \\ (2n-1)i - 2j - 7, & \text{if } 2 \leq i \leq k \text{ and } 2 \leq j \leq r-3 \\ (2n-1)i - 2j - 13, & \text{if } 2 \leq i \leq k \text{ and } r-2 \leq j \leq n-6 \\ (2n-1)i - n - 4, & \text{if } 2 \leq i \leq k \text{ and } j = n-5 \\ (2n-1)i + 2j - 2n, & \text{if } 2 \leq i \leq k \text{ and } n-4 \leq j \leq n-1 \\ (2n-1)k, & \text{if } i = k \text{ and } j = n. \end{cases}$$

It can be easily proved using mathematical induction on s as in the above theorems that the labeling $f : E(kC_n) \to \{1, 2, 3, \ldots, (2n-1)k + 1\}$ is an SVM labeling.

Example 4.10 *An SVM labeling of a Tridecagonal (C_{13}) snake with 2 blocks is shown in Figure 7.*

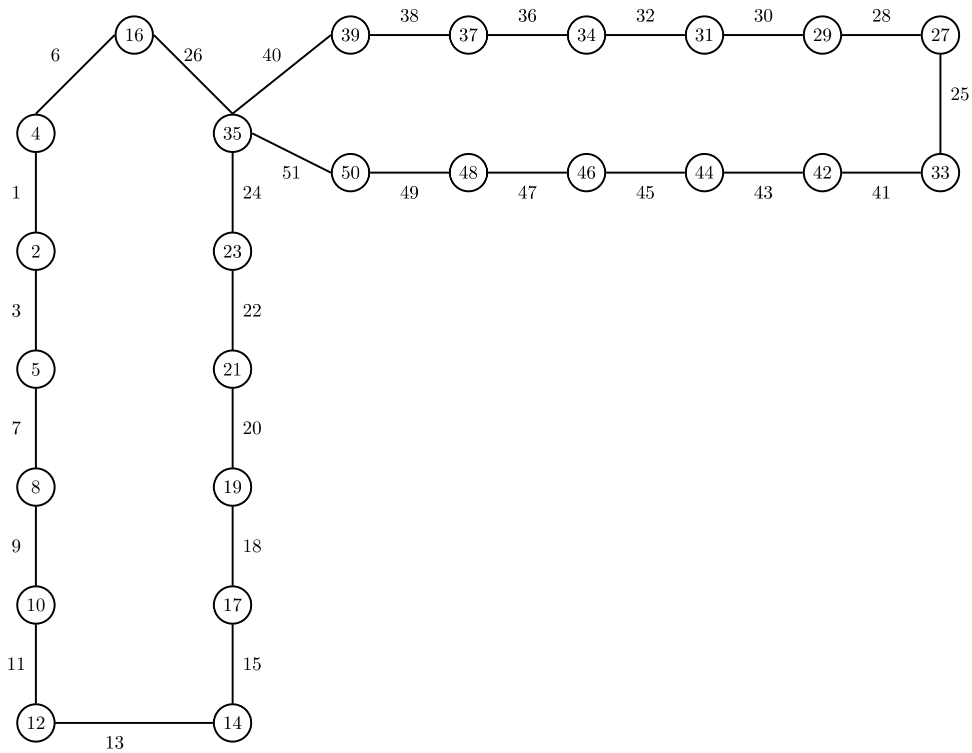

Figure 7: An SVM labeling of a Tridecagonal (C_{13}) snake with 2 blocks.

Theorem 4.11 *Let kC_n be a cyclic snake with k blocks of $C_n, n \geq 10$ and $n \equiv 2(mod\ 4)$. Then kC_n is a Super Vertex Mean graph.*

Proof: Let kC_n be a cyclic snake with k blocks of $C_n, n \geq 10$ and $n \equiv 2(mod\ 4)$.

Let $n = 2r$, and $r = 2s + 1$ so that $n = 4s + 2$.

Let $V(kC_n) = \{v_{i,j}; 1 \leq i \leq k, 1 \leq j \leq n\}$ and $E(kC_n) = \{e_{i,j} = v_{i,j}v_{i,j+1}$ & $e_{i,n} = v_{i,n}v_{i,1}; 1 \leq i \leq k, 1 \leq j \leq n-1\}$.

Note that $v_{i,n} = v_{i+1,1}$ for $1 \leq i \leq k-1$, and we refer this vertex as $v_{i,n}$ throughout this proof.

Now, $p = (n-1)k + 1$ and $q = nk$ and $p + q = (2n-1)k + 1$.

Define $f : E(kC_n) \to \{1, 2, 3, \cdots, (2n-1)k+1\}$ as follows,

$$f(e_{i,j}) = \begin{cases} 2n-2j-2, & \text{if } i=1, \text{ and } 1 \le j \le r-4 \\ 2n-2j-3, & \text{if } i=1, \text{ and } r-3 \le j \le n-6 \\ 3n-3j-9, & \text{if } i=1, \text{ and } n-5 \le j \le n-4 \\ 1, & \text{if } i=1, \text{ and } j=n-3 \\ 7, & \text{if } i=1, \text{ and } j=n-2 \\ 4n-2j-2, & \text{if } i=1, \text{ and } n-1 \le j \le n \\ 2n-1, & \text{if } i=2, \text{ and } j=1 \\ 2n+2j-2, & \text{if } i=2, \text{ and } 2 \le j \le r-s \\ 2n+2j-1, & \text{if } i=2, \text{ and } r-s+1 \le j \le n-1 \\ (2n-1)i+3, & \text{if } 2 \le i \le k-1 \text{ and } j=n \\ (2n-1)i+2j-2n-1, & \text{if } 3 \le i \le k \text{ and } 1 \le j \le 2 \\ (2n-1)i+2j-2n, & \text{if } 3 \le i \le k \text{ and } 3 \le j \le r-s \\ (2n-1)i+2j-2n+1, & \text{if } 3 \le i \le k \text{ and } r-s+1 \le j \le n-1 \\ (2n-1)k+1, & \text{if } i=k \text{ and } j=n. \end{cases}$$

And, the induced vertex labels are as follows:

$$f^v(v_{i,j}) = \begin{cases} 2n-2j-1, & \text{if } i=1, \text{ and } 1 \le j \le r-3 \\ 2n-2j-2, & \text{if } i=1, \text{ and } r-2 \le j \le n-5 \\ 5, & \text{if } i=1, \text{ and } j=n-4 \\ 8+2j-2n, & \text{if } i=1, \text{ and } n-3 \le j \le n-2 \\ n+4, & \text{if } i=1, \text{ and } j=n-1 \\ (2n-1)i+r+1, & \text{if } 1 \le i \le k-1 \text{ and } j=n \\ 2n+2j-1, & \text{if } i=2, \text{ and } 2 \le j \le r-s \\ 2n+2j-2, & \text{if } i=2, \text{ and } r-s+1 \le j \le n-1 \\ (2n-1)i+2n+2, & \text{if } 3 \le i \le k \text{ and } j=2 \\ (2n-1)i+2j-2n-1, & \text{if } 3 \le i \le k \text{ and } 3 \le j \le r-s \\ (2n-1)i+2j-2n, & \text{if } 3 \le i \le k \text{ and } r-s+1 \le j \le n-1 \\ (2n-1)k, & \text{if } i=k \text{ and } j=n. \end{cases}$$

It can be easily proved using mathematical induction on s as in the above theorems that the labeling $f : E(kC_n) \to \{1, 2, 3, \ldots, (2n-1)k+1\}$ is an SVM labeling.

Example 4.12 *An SVM labeling of a Tetradeagonal* (C_{14}) *snake with* 3 *blocks is shown in Figure* 8.

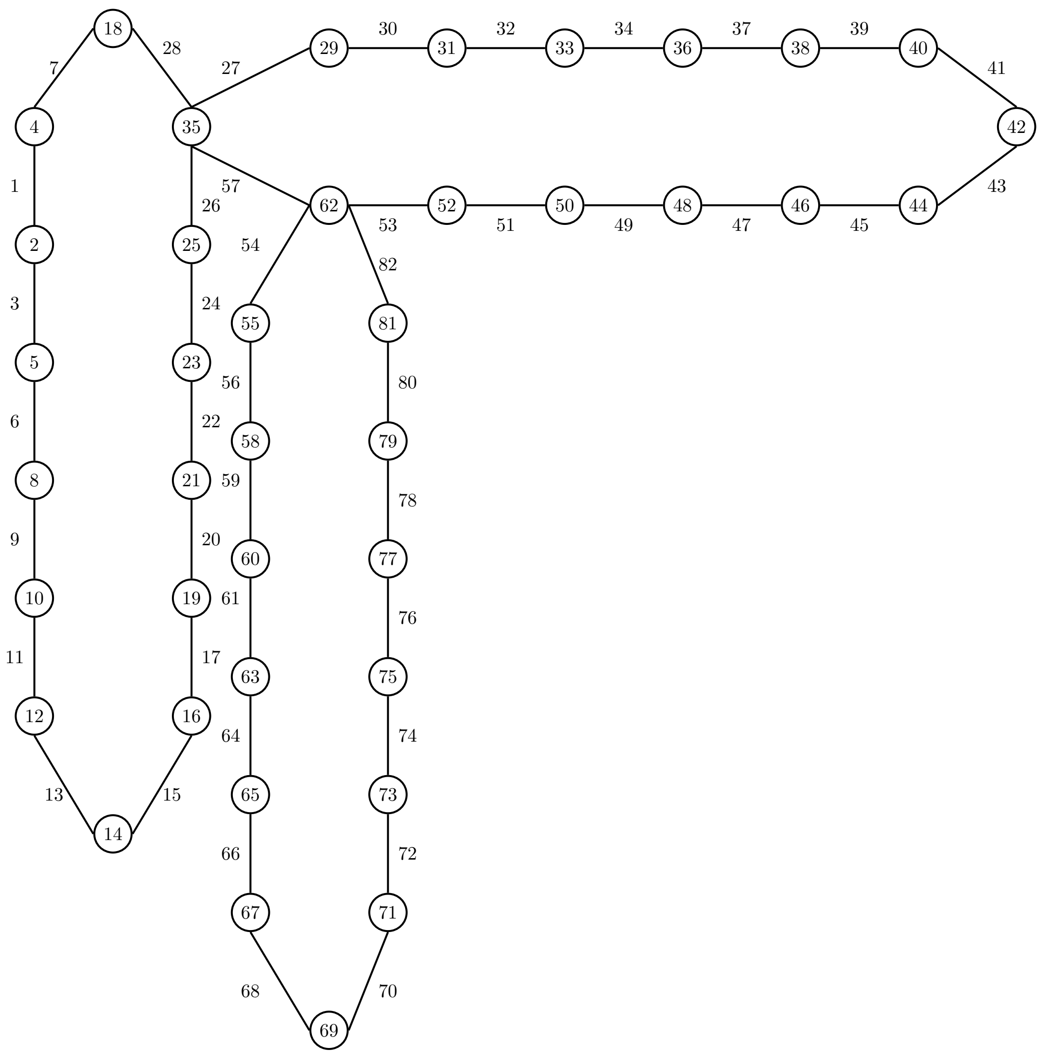

Figure 8: An SVM labeling of a Tetradecagonal (C_{14}) snake with 3 blocks.

5 Conclusion

In this paper, we have proved that all the cyclic snakes are Super Vertex Mean graphs, provided each s_i on the string $s_1, s_2, s_3, \cdots, s_{k-2}$ which is used to represent a kC_n cycle is equal to 1. This s_i is the distance between i^{th} and $i+1^{th}$ cut vertices along the path, P, where P is the path of minimum length that contains all the cut vertices of a kC_n – snake, starting from one extreme and is taken from $S_n = \{1, 2, 3, \cdots, \lfloor \frac{n}{2} \rfloor\}$.

In the case of Super Mean Labeling, the vertex analogue of SVM, it was easy to obtain a general formula for cyclic snakes represented the string $s_1, s_2, s_3, \cdots, s_{k-2}$, where each s_i need not be equal to 1. This is because when we calculate the induced edge label by finding the average of the labels of two vertices which are the end points of the respective edge, we need to only consider those two vertices. Therefore the average remains the same as in the case of cycles.

But for Super Vertex Mean labeling, when we find the induced vertex labeling of the connecting vertices of a cyclic snake we have to consider four edges that are incident on those vertices to get the average. Thus it becomes pretty difficult to obtain a general formula for cyclic snakes represented the string $s_1, s_2, s_3, \cdots, s_{k-2}$, where each s_i need not be equal to 1. Another possibility in this area is to find out SVM labelings of cyclic graphs whose each s_i is equal, and need not be equal to 1, as we have proved in this paper.

Another possibility that emerges for further study is that we try to explore the SVM labeling of kC – snakes, which are defined as connecting graphs in which each of the k many blocks is isomorphic to a cycle C_n for some n and the block - cut point graph is a path. As in the case of kC_n – snakes, a kC – snake too can be represented by a string of integers, $s_1, s_2, \cdots, s_{k-2}$. Thus, it is still an open problem to label a kC – snake which has either the same value or different values for each s_i.

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