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Research Article

On Problem of Tricomi Equation with Smooth Boundry Data

Mary Chern

8-53 Ontario Road,Welland, On L3B 5C2, Canada

Corresponding author: Mary Chern; E-mail: w-cb@live.ca

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Abstract: In this paper, a boundary problem for the tricomi equation is studied. These results may be helpful in the study of similar problems. We will estimate the solutions and obtain the existence. Further applications of such theorems will be expected.

Keywords: Degenerate.

1. Introduction

In this part, we will introduce the equation and some notations for convenience.

Consider the equation:

(1) $yu_{xx} + u_{yy} = 0$, in Ω,
(2) $u = \tilde{g}$ on $\partial\Omega$,

Ω is the boundary of the region, with part of the boundary composed of a subset of the x −axis, so the equation is degenerate there. On the rest of the boundary $y > 0$. The boundary is Lipschitz continuous.

We want to obtain some estimates first, for simplicity, let us assume $u \in C^2$, then we may apply the divergence theorem for such a solution to (1)(2),

$$\int div\,\vec{u} = \oint \vec{u} \cdot \vec{n} ds,$$

Note that if we for simplicity find an extension of $\tilde{g}$, such that, on $\partial\Omega$, $g = \tilde{g}$,of course, we assume that $g \in C^2$, then let $v = u - g$, which is equivalent roughly to the following:

$yu_{xx} + u_{yy} = \mathrm{f}$, in Ω, Where we used the same symbol in the left hand side, and the right hand side is a given function. We know that

$$\int (yu_x v_x + u_y v_y) = -\int fv = \int (yg_x v_x + g_y v_y),$$

This formula defines a weak solution, and, we claim

There is a weak solution in

$$\|u\|_H^2 = \int (yu_x^2 + u_y^2).$$

That concludes our study on the weak solution of (1) (2), the conclusion follows from Rietz theorem.

$$\|u\|_H \leq c\big(\|g\|_{C_1}\big).$$

In a Hilbert space, every bounded functional can be written in the form:

$$(u, v)_H, u, v \in H.$$

Proof: Due to the fact that the space here is separable, we let $u = \sum_i l(e_i)e_i, \{e_i\}$ is a basis, s.t.$e_i \cdot e_j = \delta_{ij}$,   Since we have

For $a \in H, a = \sum_i a_i\, e_i$, then,

$l(a) = \sum_i a_i\, l(e_i), |l(a)| \leq \|l\|\|a\|$, for any $a \in H$, let

$a = \sum_{i=1}^n l(e_i)e_i$, therefore

$$\sum_i l(e_i)^2 < \infty.$$

Comparable to the Poincare theorem, we can establish this theorem:

For $u \in H, \int u^2 \leq c\|u\|_{H.}^2$

Proof: First we let $u \in C_0^\infty(\Omega)$, then assume that Ω is a $x -$ domain, that is,

$$a \leq x \leq b, Y_1(x) \leq y \leq Y_2(x), \forall x \in [a, b],$$

$$u(x, y) = \int_{Y_1}^{y} u_t\,(x, t)dt,$$

$$\int u^2 = \int_a^b dx \int_{Y_1}^{Y_2} u^2 dy \ll c \int u_y^2,$$

it holds for $u \in H$ by completion.

If $u \in C_0^\infty$, let us claim that it satisfies:

$$yu_{xxx} + u_{xyy} = f_x,$$

and we conclude that

$$\int \left(yu_{xx}^2 + u_{xy}^2\right) \le c(\|g\|_2).$$

2. Regularity of the Solution

Let $u = u(x)$ be a solution in H, then in order to study the regularity, consider $u_\varepsilon = uj_\varepsilon(y), j$ is the mollifier at x axis.

$$yu_{\varepsilon xx} + u_{\varepsilon yy} = fj_\varepsilon + 2j_\varepsilon' u_y + j_\varepsilon'' u,$$

The right-hand side $\in L_2, u_\varepsilon$satisfies this equivalent weak formulation

$$\int (yu_x v_x + u_y v_y) = -\int (fj_\varepsilon + 2j_\varepsilon' u_y + j_\varepsilon'' u)v, v \in C_0^\infty\{y > \varepsilon\}.$$

For$v \in C_0^\infty\{y > \varepsilon\}, \int(yv_x^2 + v_y^2) \ge c\int(v_x^2 + v_y^2), c > 0, u_\varepsilon \in H_1^0$, furthermore,

Assuming$g \in C^{2,1}$, we claim that

$\int\left(u_{\varepsilon xx}^2 + u_{\varepsilon xy}^2\right) \le c(g,\varepsilon)\|u\|_H^2,$

Firstly, assume that u_ε is in C_0^∞ then it satisfies this weak formulation:

$\int(yu_{xx}v_x + u_{xy}v_y) = \int(l.r.t. + hu_{xy}v), h$ is a known function, set $u = u_{\varepsilon,} v = u_{\varepsilon x,}$, then it follows by the generalized Cauchy-Schwartz inequality,

$\int|ab| \le \int \varepsilon a^2 + \frac{1}{\varepsilon}b^2, \varepsilon > 0$, set $a = u_{xy}$, $b = u_x$, we make ε small enough so that $\|h\|_C \varepsilon < \frac{1}{2}$.

Equivalently,

$$\int (u_{\varepsilon xx}^2 + u_{\varepsilon xy}^2) \le c(g,\varepsilon)\|u\|_{H,}^2$$

Similarly, such estimation holds for$\int u_{\varepsilon yy}^2$,

It follows that inside Ω, $u \in C$.

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